The structure of labor market flows[*]

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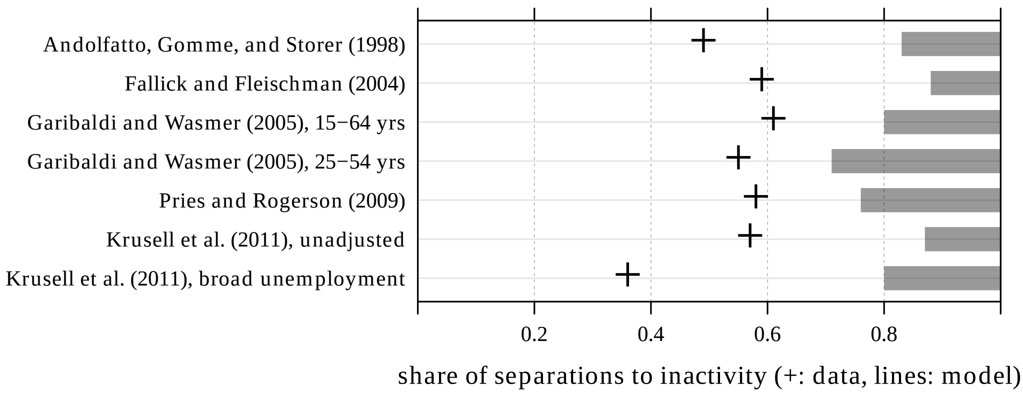
Abstract

We show that a general class of frictional labor market models with a participation margin and an individual-specific state can only match labor market transition rates within a certain range, which we characterize analytically. Transition rates in the data are outside the range the model can match, which explains the failure of previous papers to calibrate to these flows. We also examine whether extending the model can bring it closer to the data, and find that endogenous search intensity and state-dependent separation rates do not help, but misclassification, persistently inactive workers, and modifications of the productivity process such as learning on the job can match the gross flows.

Graphical abstract



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1 Introduction

The last ten years have seen the emergence of comprehensive models for the labor market, which aim to explain the individual-level dynamics of both labor supply choices and unemployment, modeling transitions between employment, unemployment, and non-participation in a frictional labor market. Recent examples are Garibaldi and Wasmer (2005) and Krusell et al. (2011): the most important addition to the previous literature is that these papers aim to pin down and explain not only stocks, but also gross labor market flows. These flows are very important for policy analysis: to the extent that they are exogenous from the perspective of an individual, such as exogenous separations, they are usually insurable to a very limited extent (if at all), and when they are the outcome of optimizing behavior, such as participation decisions, it is important to understand how they would responds to various policy measures, for example taxation and unemployment insurance.

Both Garibaldi and Wasmer (2005) and Krusell et al. (2011) calibrate to the observed labor market flows, and are able to explain them with a seemingly small discrepancy between the implications of the model and the data. This paper demonstrates that this small discrepancy is significant, and it is not something that can be improved on with a more careful calibration: a general class of simple labor market models with labor market frictions, and an individual-specific state which drives participation decisions and evolves stochastically, can only explain labor market transition rates if they are within a certain range. Examination of the data used by various papers shows that the flows are outside the range that the model can explain. Consequently, the inability of this model family to match the data stems not from the lack of enough free variables, but is a feature of *both* the model and the data: as we explain below, matching certain flows constrain the ability of the model to match other flows, and in the data the latter flows are outside the admissible range of the model.

To make things concrete, let E, U, and I denote employment, unemployment, and inactivity (non-participation),[1] respectively, and $\lambda_{\mathrm{UI}}, \lambda_{\mathrm{UE}}, \dots$ denote continuous-time transition rates between these states. The central question of this paper is the following: for a given model and 6-tuple of transition rates

$$\Lambda = (\lambda_{\mathrm{UI}}, \lambda_{\mathrm{IU}}, \lambda_{\mathrm{IE}}, \lambda_{\mathrm{UE}}, \lambda_{\mathrm{EI}}, \lambda_{\mathrm{EU}}),$$

can we find a parameterization of the model that generates Λ?

The key contribution of this paper is an analytical characterization of this question for various models, and the introduction of a particular way of calibrating to flows that allows a tractable analysis of this nonlinear problem. Figure 1 provides a stylized summary of the way we characterize the calibration of the models we examine. First, we calibrate to the flows between unemployment and inactivity, λ_{IU} and λ_{UI}, the *total* flows out of employment

$$\lambda_{\mathrm{E[IU]}} = \lambda_{\mathrm{EI}} + \lambda_{\mathrm{EU}},$$

and the job finding rates λ_{IE} and λ_{UE}. This way we match five out of the six moments we target. Then

[1] In this paper we refer to non-participants as *inactives*, because we want to avoid confusion with the non-employed in the notation in Section 2.

we examine the ratio

$$\alpha = \frac{\lambda_{\mathrm{IE}}}{\lambda_{\mathrm{E}[\mathrm{IU}]}} \tag{1}$$

as a function of the free parameters of the model, and see whether the range of α — which is, of course, a function of the other five moments we have matched — contains its counterpart in the data. If it does, then we say that a particular set of flows Λ are *admissible* for a given model.

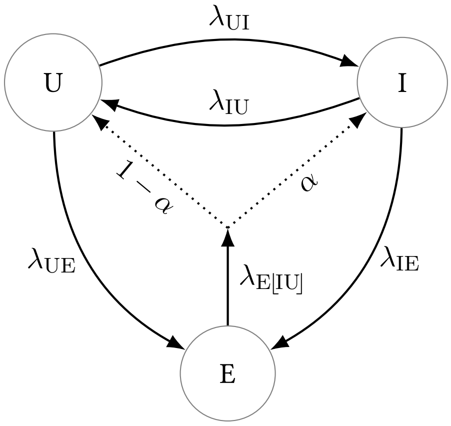

Figure 1: Calibration of labor market transition rates. We calibrate to λ_{IU}, λ_{UI}, $\lambda_{\mathrm{E}[\mathrm{IU}]} = \lambda_{\mathrm{EI}} + \lambda_{\mathrm{EU}}$, λ_{UE}, and λ_{IE} (solid lines). Then we characterize the fraction $\alpha = \lambda_{\mathrm{EI}}/\lambda_{\mathrm{E}[\mathrm{IU}]}$ (dotted lines).

For some of the models we examine, including the benchmark model of Section 2 which nests Garibaldi and Wasmer (2005) and closely approximates a special case of Krusell et al. (2011), we find that when calibrating to the other five moments, there is a lower bound α^* such that a particular α is admissible if and only if

$$\alpha^* \leq \alpha \leq 1,$$

where we characterize the lower bound α^* analytically as a function of the five other moments we calibrate to.

We briefly summarize the benchmark model and explain the intuition behind this result. The benchmark model of Section 2 is in continuous time with linear utility, the only state of a worker is individual-specific, and it determines wages and the flow utility of non-employment (eg the value of leisure, home production, and unemployment benefits). The difference of the two is the flow surplus, and it plays a key role in the labor market participation choice. Exogenous events, which can be thought of as stylized representations of shocks that change either wages or the value of non-employment, change the individual's state and thus the flow surplus. Employed workers experience exogenous separations, but may also separate endogenously after if their state and consequently their flow surplus changes.

Non-employed workers can choose to search *actively* or *passively*. Job opportunities arrive exogenously, and are either accepted or rejected by the worker, and active search results in a higher arrival rate, but entails a flow cost. Consequently, the flow surplus partitions the state space of non-employed workers into three regions: those with a high surplus for whom it is worthwhile to pay the search cost in exchange for a higher arrival rate of job offers, so they search actively (H), those who would accept a job but would not pay the search cost, and they search passively (M), and those whose flow surplus is so low that they would never accept a job (L).

We identify unemployed workers in H with active search, and the other two regions L and M with non-participation. A key feature of the model is that the five flows we calibrate to constrain the calibration of the stochastic process for the individual-specific state, which in turn determines the distribution of *employed* workers in regions M and H of the parameter space, which we call *marginal* and *non-marginal* workers, respectively. This is important because after an exogenous separation, non-marginal workers search actively, while marginal workers search passively, and this constrains the share of λ_{EI} and λ_{EU} flows in $\lambda_{\mathrm{E[IU]}}$, thus determining α in (1).

Looking at the data in various papers that calibrate to labor market flows in Section 3, we find that

$$\alpha_{\text{data}} < \alpha^*,$$

in other words when calibrating to the other five moments, α in the data is lower than the lower bound of what the model can be calibrated to and thus the flows are not admissible in the benchmark model. Mostly, this happens because in the data,

$$\lambda_{\mathrm{IU}} + \lambda_{\mathrm{E[IU]}} \ll \lambda_{\mathrm{UI}},$$

or in other words, having so many inactives relative to the unemployed requires that UI transition rates are much larger than IU rates. In theory, a large separation rate from E could alleviate this, because, since $\lambda_{\mathrm{IE}} < \lambda_{\mathrm{UE}}$, it would lead to relatively fewer marginal inactives; but of course this is not a feature of the data. We show that this discrepancy cannot be explained by small sample size, and it holds for all papers we have examined, which suggests that a crucial ingredient is missing from our models if we want to calibrate to labor market flows.

Consequently, Section 4 examines various extensions to the benchmark model. First, recognizing that the simple binary search technology in the benchmark model may be too restrictive, in Section 4.1 we endogenize search effort with a continuous variable, and show that no matter where we draw the line between inactivity and unemployment, the flows can be mapped to those of the benchmark model, and this extension does not improve its ability to match the data.

In Section 4.2 we extend the model with state-dependent separation rates: allowing for the possibility that marginal workers experience higher rates of exogenous separation compared to non-marginal workers. We think of this extension as reduced form for a model with firm- or match-specific productivity, with the idea that marginal matches are more fragile. Since this modification increases λ_{EI} flows, it actually increases α^*, moving the model further away from the data.

A crucial assumption in the result about the benchmark model is that we impose a stochastic process for the individual-specific state that is independent of the labor market status, which makes λ_{IU} and λ_{UI} rates, which are from the observation of the non-employed, constrain the share of marginal employed. Breaking the connection between the stochastic processes for the individual-specific state for the employed and non-employed could in principle alleviate the problem of the benchmark model. We show that this is indeed the case, first by examining a stylized model with permanently inactive workers in Section 4.3, then by introducing learning on the job in Section 4.4. Having permanently inactive workers who never participate in labor market flows helps because we can assume that the λ_{IU} flows we observe are the weighed average of the same flows for permanently inactive workers (for whom it is zero) and the rest of the population, for whom they are consequently larger, and larger λ_{IU} flows would lower α^*. In contrast, learning on the job works by decreasing the share of marginal employed, since by moving to a state with higher surplus they become non-marginal. We show that in theory both approaches can drive α^* to 0, and thus all possible flows Λ become admissible under either model.

Finally, in Section 4.5 we consider classification error, by allowing for the possibility that inactive or unemployed workers are misclassified into the other state in surveys, thus generating spurious flows. We introduce a general theoretical framework for calculations with such processes, which provides a mapping from observed to underlying flows in a self-consistent way. We find misclassification of inactives as unemployed *increases* the lower bound α^*, but misclassification of unemployed as inactive *decreases* α^* and brings the model closer to the data — in fact, a UI misclassification probability for each observation around 9% can bring α^* to α_{data}.

Our analysis is related to the various approaches in the literature that aim to explain participation and unemployment. As noted by Krusell et al. (2011), historically, frictionless versions of the standard growth model were mainly used to explain participation, mapping it to a choice on the labor/leisure margin: for example Hansen (1985) and Rogerson (1988), while models in the Diamond-Mortensen-Pissarides model family[2] have been used to explain unemployment with labor market frictions and the response of unemployment to aggregate fluctuations.[3] However, recognizing that satisfactory models should account for both unemployment and the participation margin, many papers incorporated the latter into frictional models of the labor market. Ljungqvist and Sargent (1998), Ljungqvist and Sargent (2007a), Alvarez and Veracierto (2000), and Veracierto (2008) are models that are similar to the one discussed in this paper along many dimensions, but they do not attempt to account for labor market flows across the three states. Coming from the other direction, Merz (1995), Andolfatto (1996), Gomes, Greenwood, and Rebelo (2001) include labor market frictions in the standard growth model, but do not distinguish unemployment and inactivity.

The two papers most closely related to this one are Garibaldi and Wasmer (2005) and Krusell et al. (2011). In particular, our model nests the structure of Garibaldi and Wasmer (2005), formalizing the reason for the discrepancy between observed and generated flows in their paper in a more general setting. What we call marginal workers they term "employment hoarding", since it results from the irreversibility of separations. Krusell et al. (2011) also focus on the flows, in a model that is essentially similar to ours except for the fact that they also allow risk aversion and saving. However, as noted in their paper, this does not have a significant effect on the flows, and thus would only complicate our analysis. Krusell et al. (2011) also argue that marginally inactive in the data should be counted as unemployed when accounting for the flows. However, this turns out to increase UI transition rates, increasing the distance between the model and the data even further. Both papers note the discrepancy between calibrated transition rates and the data, but focus on its consequences on the UI and IU flows.

In contrast to most of the literature, the models in this paper are very stylized, and we focus on

[2]See Pissarides (2000) for an introductory overview.

[3]See, for example, Haan, Ramey, and Watson (2000), Costain and Reiter (2008), Shimer (2005).

discussing the theoretical properties of models, with respect to their ability to match labor market flows. This is necessary because in order to say that a particular set of flows Λ is *not* admissible for a certain model, we need to be able to characterize the whole range of flows that are possible, which necessitates a stylized structure. The stylized models in this paper may not be directly applicable for policy questions, but are intended to serve as a useful guide on what to incorporate into models which are calibrated to labor market flows. This approach of characterizing the distance between data and models with a single scalar, and focusing on simple models to explore extensions that alleviate a puzzle was inspired by Hornstein, Krusell, and Violante (2011).

Also, this paper does not discuss the implications of the models for wages: the models are used to put structure on the observed transition rates between employment, unemployment and inactivity. This is not because we think that wages are not important, but because explaining flows themselves appears to be difficult enough — also, since only the difference of wages and non-employment utility matters for the labor market flows in the models we discuss, strong assumptions on wage processes would be needed to connect wages and labor market flows. We leave incorporating wages for future research.

Most of the results in the paper are analytical, but illustrated with calculations using empirical data. However, in order to avoid making the paper unreadable, we relegated most steps in the analytical proofs to the appendix, and only included important equations and simplified derivations in the main text. All analytical proofs and calculations have been checked using the symbolic algebra software Maxima (2014) and are available in the online Appendix A.

2 The benchmark model

In this section we introduce a model that serves as a starting point for the discussion of gross labor market transition rates. This model makes a compromise between tractability and generality: it is simple enough to allow an analytical characterization of the results, yet at the same time general enough to nest or approximate partial equilibrium features of models in the literature; in particular, the model nests the partial equilibrium features of Garibaldi and Wasmer (2005). The model could be embedded in a general equilibrium framework, for example similarly to Hornstein, Krusell, and Violante (2011, Section 1.B), but this would not add to the key results of the paper.[4]

2.1 Preferences and technology

Time is continuous, workers are risk-neutral, ex-ante homogeneous and discount at rate r.[5] We characterize the steady state equilibrium in which job offer rates are exogenous, conditional on search intensity. Workers are either *employed* or *non-employed*, and workers in the latter are categorized as either *unemployed* or *inactive* based on search activity. Since the model needs to be able to generate

[4]A general equilibrium formulation with job (vacancy) creation would just provide additional restrictions on the gross flows that the model can generate, and thus the range of labor market flows generated by a partial equilibrium model is necessarily larger than it would be for general equilibrium one. Using the latter would just complicate the formulation and distract from Lemma 1. A partial equilibrium formulation also precludes discussion of aggregate fluctuations and policy experiments, but both of those directions are outside the scope of this paper.

[5]We refer to all agents as workers, regardless of their current employment status.

apparent flows from seemingly inactive workers (ie those workers who are neither employed nor unemployed, in the sense that they do not search actively) into employment, we assume that workers who choose not to search actively also receive job offers, albeit at a lower rate.[6] Specifically, there are two search technologies available to all non-employed workers: *active search*, which entails a flow cost c with job offers arriving at rate φ_{h}, and *passive search* which requires no search effort (ie a cost of 0), and makes job offers arrive at rate φ_{m}. Naturally, $\varphi_{\mathrm{m}} < \varphi_{\mathrm{h}}$. This is a very stylized specification as it only allows a binary choice for search effort, we generalize this in Section 4.1.

Workers have an individual-specific state x that we think of as a proxy for market opportunities, family, health, and preference shocks. Krusell et al. (2008) show that it is difficult to generate gross labor market flows between inactivity and unemployment without these shocks even in a very rich model with precautionary savings, in the absence of the latter this process will be the only source of flows between unemployment and inactivity, and also, to a certain extent, from employment to non-employment. We think of changes in x as major life events that affect the difference between labor market productivity and the the opportunity cost of working, such as changes in personal relationships or family status, a major illness, and education opportunities and attainments. The individual's state x determines wages $w(x)$ and the utility flow for the nonemployed, $u(x)$, where the latter includes unemployment benefits, home production, and the value of leisure. This implies that all worker heterogeneity in the benchmark model is individual-specific, and there are no match- or firm-specific sources of wage dispersion.[7]

The state x is constant until a *change event* arrives, in which case it is redrawn from an IID distribution $x \sim F$. Change events arrive at rate γ, independently of other events and states, particularly labor market status. We think of γ as being a relatively low number, because major changes in workers' market productivity or outside options are expected to be rare. Hornstein, Krusell, and Violante (2011) refer to this kind of process as a *persistent process*, and it is commonly used to specify stochastic processes which exhibit some degree of persistence (and thus autocorrelation), yet at the same time allowing a simple characterization of steady state distributions and transition rates.

This specification is restrictive in two ways: it only allows IID distributions for x conditional on a change event, and it imposes the same process for both the employed and non-employed. It turns out that the latter has important implications for matching labor market flows, and we consider various generalizations in Sections 4.3 and 4.4.[8] At the same time, the benchmark model allows an arbitrary space for the values of x: discrete distributions, subsets of $\mathbb{R}^n$, or even combinations of the two, as long as they capture all payoff-relevant information and new values are IID conditional on a change event.

Employed workers may separate endogenously whenever they prefer non-employment to employment — this happens if they experience a change event that results in a draw of x where the difference between $w(x)$ and $b(x)$ is low. In addition, employed workers are also subject to exogenous separa-

[6]As we discuss in Section 3, even though a fraction of these transition can be explained by time aggregation (inactive workers becoming unemployed and then employed between two observations), this flow is too large in the data to be assumed away.

[7]The model in Section 4.2 can be considered a reduced-form version of extending the benchmark model with match- or firm-specific heterogeneity in a way that marginal matches would be less robust to shocks, resulting in higher exogenous separations.

[8]A previous version of this paper also had a generalization to non-IID distributions for F, which is omitted because it does not add significantly to the results but greatly complicates derivations.

tion shocks at rate σ. We assume that the separation rate is uniform and thus independent of worker surplus and history, we generalize this in Section 4.2. Table 1 summarizes the notation for parameters and endogenous objects of the benchmark model.

	parameters
$x \in \mathcal{X}$	individual-specific state and set of possible states
γ	arrival rate of change events for x
F	distribution of new x, conditional on a change event
$w(x)$	wage when employed
$b(x)$	flow value of non-employment (unemployment benefit, leisure, and home production)
$\varphi_{\mathrm{m}}, \varphi_{\mathrm{h}}$	arrival of job offers for passive and active search
c	flow cost of active search
σ	rate of exogenous separation
	equilibrium objects
$W(x)$, $N(x)$	present discounted value of employment, non-employment
$S(x)$	present discounted value of the worker's surplus
$\mathrm{L} \subset \mathcal{X}$	low surplus: no active search, non-employment preferred
$\mathrm{M} \subset \mathcal{X}$	marginal surplus: passive search, employment preferred
$\mathrm{H} \subset \mathcal{X}$	high surplus: active search
$q_\ell, q_{\mathrm{m}}, q_{\mathrm{h}}$	continuous-time rate of transition to regions L, M, H, respectively
λ_{IU}, $\lambda_{\mathrm{IE}}, \ldots$	observed transition rates between **I**nactivity, **U**nemployment, and **E**mployment
$\lambda_{\mathrm{E[IU]}}$	transition rate out of employment, $\lambda_{\mathrm{EI}} + \lambda_{\mathrm{EU}}$
ν	share of marginal workers among the inactive
μ	share of marginal workers among the employed
α	share of λ_{EI} in $\lambda_{\mathrm{E[IU]}}$

Table 1: Notation for the benchmark model of Section 2. Notation recycled for extensions.

2.2 Value and policy functions

Let $N(x)$ and $W(x)$ denote the current present value of being non-employed or employed, respectively, with individual-specific state x. The continuous time Hamilton-Jacobi-Bellman equations are

$$rN(x) = b(x) + \max\Big\{\underbrace{\varphi_{\mathrm{m}} \max\big\{W(x) - N(x), 0\big\}}_{\text{passive search}}, \underbrace{-c + \varphi_{\mathrm{h}}\big(W(x) - N(x)\big)}_{\text{active search}}\Big\} + \underbrace{\gamma\, \mathbf{E}_{x'}\big[N(x') - N(x)\big]}_{\text{state change}} \tag{2}$$

$$rW(x) = w(x) + \underbrace{\sigma\big(N(x) - W(x)\big)}_{\text{exogenous separation}} + \underbrace{\gamma\, \mathbf{E}_{x'}\Big[\max\big\{W(x'), N(x')\big\} - W(x)\Big]}_{\text{state change, maybe endogenous separation}} \tag{3}$$

Equation (2) states that a nonemployed worker receives benefits (which are function of x), and can choose between passive and active search. For the former, offers are only accepted when working is preferable to non-employment, for the latter, the formulation above anticipates that when agents choose to search actively, they accept the job they find. The exogenous state change always leaves a nonemployed worker nonemployed, and thus it generates flows between inactivity and unemployment.

For an employed worker, (3) shows that the flow payoff is the wage, and the two possible transitions are exogenous separations and state changes. Exogenous separation always moves the worker into non-employment, while changes of the individual state can result in endogenous separation if the worker ends up in a state where $w(x)$ is low compared to $b(x)$.

It can be shown that the system (2) and (3) has a unique solution using a standard contraction argument, consequently the model parameters determine the policy function for the binary search intensity. However, as usual in this model family, it is more convenient to analyze the model in terms of the worker's surplus

$$S(x) = W(x) - N(x)$$

Rewrite (2) and (3) in terms of the surplus as

$$rN(x) = b(x) + \max(\varphi_{\mathrm{m}} S(x)^{+}, \varphi_{\mathrm{h}} S(x) - c) + \gamma\Big(\mathbf{E}_{x'}\left[N(x')\right] - N(x)\Big) \tag{4}$$

$$rW(x) = w(x) - \sigma S(x) + \gamma\Big(\mathbf{E}_{x'}\left[S(x')^{+}\right] + \gamma\, \mathbf{E}_{x'}\left[N(x')\right] - W(x)\Big) \tag{5}$$

where $S(x)^{+} = \max\big(0, S(x)\big)$.

Introduce the flow surplus $s(x) = w(x) - b(x)$, then combine (4) and (5) into

$$(r+\sigma)S(x) = s(x) - \underbrace{\max(\varphi_{\mathrm{m}} S(x)^{+}, \varphi_{\mathrm{h}} S(x) - c)}_{\text{opportunity cost of not searching}} + \underbrace{\gamma\Big(\mathbf{E}_{x'}\left[S(x')^{+}\right] - S(x)\Big)}_{\text{change event}} \tag{6}$$

Equation (6) characterizes the worker's surplus in terms of the model parameters. The effective discount rate on the left hand side is the subjective discount rate r and the exogenous separation rate σ, as exogenous separations terminate the match. On the right hand side, $s(x) = w(x) - b(x)$ is the flow payment for a surplus: this demonstrates that only the difference of market productivity and the opportunity cost of working (such as the value of leisure, home production, or unemployment benefits) matters for the determination of the surplus and consequently the search policy; in this model, wages and flows are orthogonal features of the data, and information about one does not help in identifying the other without additional restrictions on processes. The second term on the right hand side is the opportunity cost of not searching, either actively or passively. The last term is for the changes in surplus: since the worker will terminate the match whenever $S(x') < 0$, the surplus cannot be below 0 for a new draw x'.

When $S(x) < 0$, the worker would not accept a job anyway, and thus defaults to passive search. Otherwise, the worker compares the search cost c to the gain from a higher job finding rate $(\varphi_{\mathrm{h}} - \varphi_{\mathrm{m}})S(x)$, and chooses active or passive search accordingly. Consequently, comparing $c/(\varphi_{\mathrm{h}} - \varphi_{\mathrm{m}})$ to $S(x)$ partitions $\mathcal{X}$ into three regions which characterize the policy function and are crucial for the determination of flows:

$$\begin{aligned} L &= \{s : S(x) < 0\} \\ M &= \{s : 0 \le S(x) < c/(\varphi_{\mathrm{h}} - \varphi_{\mathrm{m}})\} \\ H &= \{s : c/(\varphi_{\mathrm{h}} - \varphi_{\mathrm{m}}) \le S(x)\} \end{aligned}$$

We choose these regions for mapping worker search behavior to the data. In region $L \subset \mathcal{X}$ (*low surplus*), nonemployed workers do not search actively, and if they encounter a job, they choose to remain nonemployed because their surplus from the job would not be positive, while in region $M \subset \mathcal{X}$ (middle or *marginal surplus*), nonemployed workers still do not search actively because their surplus does not justify the cost, but would accept a job if they were offered one. We assume that survey data would record these workers as *inactive*, or out of the labor force. We call non-employed in region M *marginal inactives*.

In the region $H \subset \mathcal{X}$ (*high surplus*), nonemployed workers search actively, and thus from now on we assume that they are recorded in survey data as *unemployed*. We use ℓ, m, and h as subscripts for notation below, always referring to the respective region.

Even though only nonemployed workers have a nontrivial choice in this model, when we account for the distributions it is important to also distinguish employed workers based on the partition above. Naturally, there are no employed workers in L, since employed workers ending up in this region after a change event always quit their job, separating endogenously. In contrast, employed workers ending up in region M after a change event do not quit, but they would not search actively if they experienced exogenous separation. For this reason, we call them *marginal employed*.

2.3 Latent and observed flows

Let E, U, and I denote employment, unemployment, and inactivity (non-participation) in survey data.[9] The state of a worker is $(x, \{\text{employment}, \text{non-employment}\})$, which is mapped to E, U, and I as discussed above. Let $\lambda_{\mathrm{UI}}, \lambda_{\mathrm{UE}}, \ldots$ denote continuous-time transition rates from U to I, U to E, etc. We now map model parameters to observed flows.

Since only unemployed find jobs at rate φ_{h}, it is straightforward that

$$\lambda_{\mathrm{UE}} = \varphi_{\mathrm{h}} \tag{7}$$

All inactive workers transition into region H at the same rate

$$\lambda_{\mathrm{IU}} = \gamma \int_{x' \in \mathrm{H}} dF(x') \equiv q_{\mathrm{h}} \tag{8}$$

where we have defined q_{h} as the product of the arrival rate of the change event, multiplied by the probability that $x' \in \mathrm{H}$, since draws with $x' \in \mathrm{L} \uplus \mathrm{M}$ would not result in an observable transition from

[9] Several papers use N for non-participation, in this paper we use I to avoid confusion with non-employment.

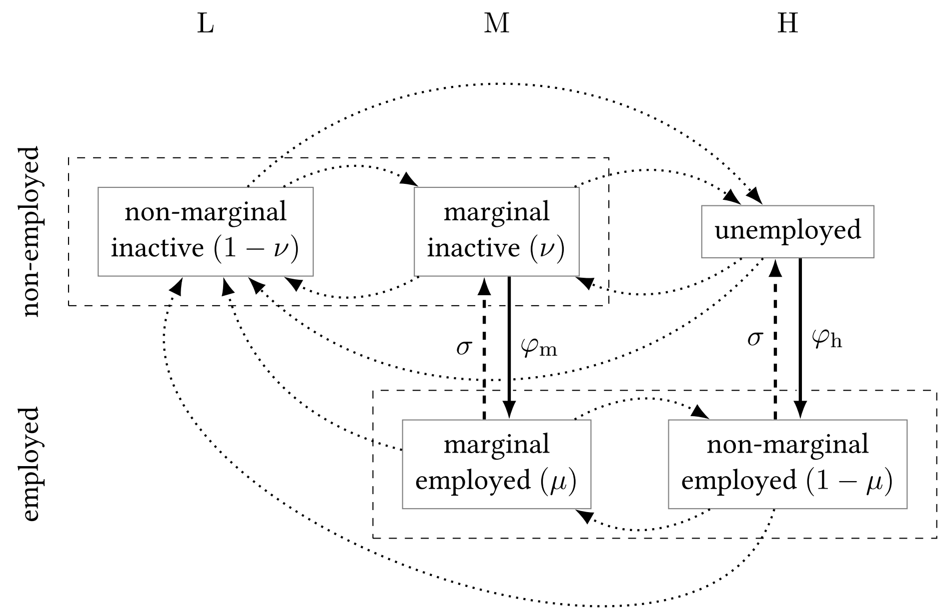

Figure 2: Latent and observed flows. Dotted arrows correspond changes in x. When these happen between L and M for nonemployed workers (both of which are counted as inactive) or M and H for employed workers, they are not observed as transitions in a dataset with three states, otherwise they show up as UI, IU, or EI transitions. Dashed arrows are exogenous separations (EI or EU, depending on whether x is in M or H), while solid arrows correspond to job finding (IE or UE), similarly depending on x.

I to U. Similarly, defining

$$q_\ell \equiv \gamma \int_{x' \in \mathrm{L}} dF(x') \qquad \text{and} \qquad q_\mathrm{m} \equiv \gamma \int_{x' \in \mathrm{M}} dF(x') \tag{9}$$

allows us to write

$$\lambda_\mathrm{UI} = q_\ell + q_\mathrm{m}. \tag{10}$$

For the other three transitions — $\lambda_\mathrm{IE}, \lambda_\mathrm{EU}, \lambda_\mathrm{EI}$ — we have to keep track of the distribution of workers, but only with respect to the partition $\mathcal{X} = \mathrm{L} \uplus \mathrm{M} \uplus \mathrm{H}$. This is because conditional on a change event, shocks to x are IID, and thus once we know that the worker is in a particular region of the state space, we know which observed transition to map to between E, U, and I. Overall, there are six possible combinations of $(\mathrm{L}, \mathrm{M}, \mathrm{H})$ and employment status, but only five of them have any mass in the steady state since there are no employed workers in L.

For analytical convenience, we characterize this distribution with the *total* of non-employed and employed workers in each region $m_\ell, m_\mathrm{m}, m_\mathrm{h}$, and the mass of employed workers $e_\mathrm{m}, e_\mathrm{h}$, where the subscript refers to the region. Then the mass of marginal inactives is $m_\mathrm{m} - e_\mathrm{m}$, and the mass if unemployed is $m_\mathrm{h} - e_\mathrm{h}$.

The steady state flow balance equations for the first three are

$$m_\ell \cdot (q_\mathrm{m} + q_\mathrm{h}) = (m_\mathrm{m} + m_\mathrm{h}) \cdot q_\ell \tag{11}$$

$$m_\mathrm{m} \cdot (q_\ell + q_\mathrm{h}) = (m_\ell + m_\mathrm{h}) \cdot q_\mathrm{m} \tag{12}$$

$$m_\mathrm{h} \cdot (q_\ell + q_\mathrm{m}) = (m_\ell + m_\mathrm{m}) \cdot q_\mathrm{h} \tag{13}$$

$$m_\ell + m_\mathrm{m} + m_\mathrm{h} = 1 \tag{14}$$

In each equation, the left hand side shows the outflows, while the right hand side shows the inflows. For example, in (11), workers transition from L to M and H with rates q_m and q_h, respectively, while workers flow into L from both of the latter regions at rate q_ℓ. Because of symmetry, it is easy to see that the solution to the system (11)–(14) is

$$m_\ell = \frac{q_\ell}{q_\ell + q_\mathrm{m} + q_\mathrm{h}} \qquad m_\mathrm{m} = \frac{q_\mathrm{m}}{q_\ell + q_\mathrm{m} + q_\mathrm{h}} \qquad m_\mathrm{h} = \frac{q_\mathrm{h}}{q_\ell + q_\mathrm{m} + q_\mathrm{h}}$$

For e_m and e_h, the steady state flow balance equations are

$$e_\mathrm{m} \cdot (q_\ell + q_\mathrm{h} + \sigma) = e_\mathrm{h} \cdot q_\mathrm{m} + (m_\mathrm{m} - e_\mathrm{m}) \cdot \varphi_\mathrm{m} \tag{15}$$

$$e_\mathrm{h} \cdot (q_\ell + q_\mathrm{m} + \sigma) = e_\mathrm{m} \cdot q_\mathrm{h} + (m_\mathrm{h} - e_\mathrm{h}) \cdot \varphi_\mathrm{h} \tag{16}$$

In (15), the left hand side shows the outflow of workers from marginal employment because of exogenous separation (σ), endogenous separation (q_ℓ), and transition into H (q_h). On the right hand side, we see the inflows from non-marginal employment due to change events (q_m), and job finding by marginal inactives (φ_m). *Mutatis mutandis*, (16) is interpreted similarly.

Let's define the fraction of marginal inactives as

$$\nu = \frac{\text{marginal inactives}}{\text{all inactives}} = \frac{m_{\mathrm{m}} - e_{\mathrm{m}}}{m_\ell + m_{\mathrm{m}} - e_{\mathrm{m}}} \tag{17}$$

Since only marginal inactives find jobs, this allows us to write the job finding rate of all inactives as

$$\lambda_{\mathrm{IE}} = \nu \varphi_{\mathrm{m}} \tag{18}$$

Similarly, we define the fraction of marginal employed as

$$\mu = \frac{\text{marginal employed}}{\text{all employed}} = \frac{e_{\mathrm{m}}}{e_{\mathrm{m}} + e_{\mathrm{h}}} \tag{19}$$

Now we consider separations from employment, into unemployment and inactivity. When employed workers experience exogenous separations, they transition into inactivity or unemployment, depending on whether they are marginal. So the observed transition rate from employment to unemployment is

$$\lambda_{\mathrm{EU}} = \frac{\text{marginal employed} \cdot 0 + \text{non-marginal employed} \cdot \sigma}{\text{all employed}} = \frac{e_{\mathrm{h}}}{e_{\mathrm{m}} + e_{\mathrm{h}}} \cdot \sigma = (1 - \mu)\sigma \tag{20}$$

In addition to exogenous separations, all employed workers transition to inactivity when they get a change event with $x' \in \mathrm{L}$. Similarly to the argument in (20), the observed gross transition rate from E to I is

$$\lambda_{\mathrm{EI}} = \mu\sigma + q_\ell \tag{21}$$

2.4 Admissible gross flows

We implement the approach outlined in Section 1, by matching the transition rates λ_{UI}, λ_{IU}, λ_{IE}, λ_{UE}, and the total separation rate $\lambda_{\mathrm{E[IU]}}$. The model has six parameters: $q_\ell, q_{\mathrm{m}}, q_{\mathrm{h}}, \sigma, \varphi_{\mathrm{m}}, \varphi_{\mathrm{h}}$, all of which have to be nonnegative, and furthermore $\varphi_{\mathrm{m}} < \varphi_{\mathrm{h}}$ has to hold. This means that matching five moments leaves us one free parameter, and it turns out to be most convenient to choose q_ℓ.

First, note that by adding (20) and (21),

$$\lambda_{\mathrm{E[IU]}} = \sigma + q_\ell$$

Intuitively, all separations happen either because of an exogenous shock (rate σ), or a change in the individual-specific state x which puts the worker in the L region, which happens at rate q_ℓ (cf (9)). This gives us

$$\sigma = \lambda_{\mathrm{E[IU]}} - q_\ell \tag{22}$$

and the restriction $q_\ell \leq \lambda_{\mathrm{E[IU]}}$ because σ has to be nonnegative. Also, notice that from (7), (8), and (10),

$$\varphi_{\mathrm{h}} = \lambda_{\mathrm{UE}} \tag{23}$$

$$q_{\mathrm{m}} = \lambda_{\mathrm{UI}} - q_\ell \tag{24}$$

$$q_{\mathrm{h}} = \lambda_{\mathrm{IU}} \tag{25}$$

From now on, we assume that

$$q_\ell \leq \min(\lambda_{\mathrm{UI}}, \lambda_{\mathrm{E[IU]}})$$

This leaves φ_{m}, which we can calibrate using (18). However, since the share of marginal inactives ν is an endogenous quantity which depends on model parameters in a nonlinear way, this turns out involve quite a bit of algebra, with little additional intuition, so we relegate this to the appendix and present a simplified derivation for looser bounds on α, which are quantitatively similar to the exact bound.

First, note that from (1), (21), and (22),

$$\alpha(q_\ell) = \frac{\lambda_{\mathrm{EI}}}{\lambda_{\mathrm{E[IU]}}} = \frac{\mu(q_\ell)\sigma(q_\ell) + q_\ell}{\lambda_{\mathrm{E[IU]}}} = \mu(q_\ell) + \big(1 - \mu(q_\ell)\big)\frac{q_\ell}{\lambda_{\mathrm{E[IU]}}} \tag{26}$$

where both α and μ are functions of the free parameter q_ℓ when matching the other five moments. Since $\lambda_{\mathrm{E[IU]}}$ is matched to the data, (26) shows that q_ℓ changes α via two channels: *directly* and via μ. The direct effect makes α increasing in q_ℓ, since $0 \leq \mu \leq 1$. The intuition behind this is simple: as the change to individual-specific state to $x' \in \mathrm{L}$ occurs with higher probability, EI flows become larger, since workers in L are inactive.

As we show below in Lemma 1, μ is decreasing in q_ℓ, but the direct effect always dominates, and thus $\alpha(q_\ell)$ is increasing, but first, we derive a simplified result that is easier to understand.

From (15),

$$e_{\mathrm{m}} \cdot (q_\ell + q_{\mathrm{h}} + \sigma) \geq e_{\mathrm{h}} \cdot q_{\mathrm{m}} \tag{27}$$

since $(m_{\mathrm{m}} - e_{\mathrm{m}})\varphi_{\mathrm{m}} \geq 0$. Using the definition of μ (19), and the calibrating equations (22), (24) and (25), this implies that

$$\mu \geq \frac{q_{\mathrm{m}}}{q_\ell + q_{\mathrm{m}} + q_{\mathrm{h}} + \sigma} = \frac{\lambda_{\mathrm{UI}} - q_\ell}{\lambda_{\mathrm{UI}} + \lambda_{\mathrm{IU}} + \lambda_{\mathrm{E[IU]}} - q_\ell} \equiv \underline{\mu}(q_\ell)$$

where we have defined a lower bound $\underline{\mu}(q_\ell)$ on $\mu(q_\ell)$. Similarly, since α is increasing in μ, we can define a lower bound

$$\underline{\alpha}(q_\ell) = \underline{\mu}(q_\ell) + \big(1 - \underline{\mu}(q_\ell)\big)\frac{q_\ell}{\lambda_{\mathrm{E[IU]}}} \qquad \text{such that} \qquad \alpha(q_\ell) \geq \underline{\alpha}(q_\ell).$$

Now using simple algebra, it is easy to show that

$$\underline{\alpha}'(q_\ell) = \frac{(\lambda_{\mathrm{IU}} + \lambda_{\mathrm{E[IU]}})(\lambda_{\mathrm{UI}} + \lambda_{\mathrm{IU}})}{\lambda_{\mathrm{E[IU]}}(\lambda_{\mathrm{E[IU]}} + \lambda_{\mathrm{IU}} + \lambda_{\mathrm{UI}} - q_\ell)^2} > 0$$

And thus, since $\alpha(q_\ell) \geq \underline{\alpha}(q_\ell) \geq \underline{\alpha}(0) = \underline{\mu}(0)$,

$$\alpha(q_\ell) \geq \frac{\lambda_{\mathrm{UI}}}{\lambda_{\mathrm{UI}} + \lambda_{\mathrm{IU}} + \lambda_{\mathrm{E[IU]}}} \tag{28}$$

Having obtained a lower bound on α, it is important to mention that we can trivially drive α up to 1, and thus make all separations go to inactivity. This can be done by making $q_\ell = \lambda_{\mathrm{E[IU]}}$, from (26) this implies $\alpha = 1$. Thus α can always be made arbitrarily large within the $[0, 1]$ interval, and there is no need to discuss upper bounds in this paper.

The lemma below shows that if we don't rely on loose bounds like (27), but also calibrate φ_{m} using (18), we can obtain exact bounds.

Lemma 1 (Bounds for $\alpha(q_\ell)$ in the benchmark model)**.** *When the benchmark model is calibrated to* λ_{IU}, λ_{UI}, λ_{IE}, λ_{UE}, *and* $\lambda_{\mathrm{E[IU]}}$,

1. $\alpha(q_\ell)$ *is strictly increasing in* q_ℓ,
2. *and has the lower bound*

$$\alpha^* = \alpha(0) = \frac{\lambda_{\mathrm{UI}}}{\lambda_{\mathrm{UI}} + \lambda_{\mathrm{IU}} + \lambda_{\mathrm{E[IU]}}(1 - \Delta)} \tag{29}$$

where

$$\Delta = \frac{\lambda_{\mathrm{IE}}(\lambda_{\mathrm{E[IU]}} + \lambda_{\mathrm{IU}} + \lambda_{\mathrm{UE}}) + \lambda_{\mathrm{IE}}\lambda_{\mathrm{UI}}}{\lambda_{\mathrm{IE}}(\lambda_{\mathrm{E[IU]}} + \lambda_{\mathrm{UE}} + \lambda_{\mathrm{UI}}) + \lambda_{\mathrm{IU}}\lambda_{\mathrm{UE}}} \qquad \textit{where} \quad 0 < \Delta < 1 \quad \textit{since } \lambda_{\mathrm{IE}} < \lambda_{\mathrm{UE}}.$$

Proof. The proof is in the online Appendix A, here we just provide a sketch. Substitute (17) into (18), and use this to eliminate the last term of (15). Solve the resulting equation and (16) for e_{m} and e_{h}, then use (19). Substitute in (22), (23), (24), (25), then use (26). The first result obtains from differentiation, the second from setting $q_\ell = 0$. □

2.5 Discussion

We illustrate the implications of Lemma 1 with labor market transition values from Garibaldi and Wasmer (2005). We choose CPS tabulations (ages 25–54) from this paper for two reasons: first, as we will see in Section 3, their data is the closest to the model among those which we consider; second, our benchmark model nests the partial equilibrium features of the model in Garibaldi and Wasmer (2005). The monthly transition rates are

$$\lambda_{\mathrm{EU}} = 0.0083, \quad \lambda_{\mathrm{EI}} = 0.0101 \qquad \Rightarrow \qquad \lambda_{\mathrm{E[IU]}} = 0.0184$$

and

$$\lambda_{\mathrm{UE}} = 0.2561, \quad \lambda_{\mathrm{UI}} = 0.1328, \quad \lambda_{\mathrm{IU}} = 0.0461, \quad \lambda_{\mathrm{IE}} = 0.0338$$

Consequently, the loose bound of (28) is

$$\underline{\alpha}(0) = \frac{\boxed{\lambda_{\text{UI}}}}{\boxed{\lambda_{\text{UI}}} + \lambda_{\text{IU}} + \lambda_{\text{E[IU]}}} = \frac{\boxed{0.1328}}{\boxed{0.1328} + 0.0461 + 0.0184} \approx 0.67$$

Notice that

$$\boxed{0.1328} = \boxed{\lambda_{\text{UI}}} \gg \lambda_{\text{IU}} + \lambda_{\text{E[IU]}} = 0.0645$$

where we have highlighted the value of λ_{UI} to emphasize that it is much larger than the other flows. In the data,

$$\alpha_{\text{data}} = \frac{\lambda_{\text{EI}}}{\lambda_{\text{E[IU]}}} = \frac{0.0101}{0.0184} \approx 0.55$$

so clearly

$$\alpha_{\text{data}} < \underline{\alpha}(0)$$

and thus even the loose bounds we have derived *without* matching λ_{IE} and λ_{UE} are violated — consequently, these features of the model are *not* driving the results. Calculating

$$\Delta = 0.60 \qquad \Rightarrow \qquad \alpha^* = 0.713$$

using (29) in Lemma 1, we can refine the bound even further. This shows that even though λ_{IE} and λ_{UE} are not driving the result, the move α further away from the model significantly.

3 Related data and literature

Since the late 1990s there has been a growing number of papers which used frictional labor market models with a participation margin to answer policy questions, or explain cross-country or secular developments in participation and unemployment rates. In this section we review a subset of this literature with two goals in mind: first, we check if the discrepancy between the data and the model that is discussed in Section 2.4 holds in the dataset(s) used by the paper, second, to discuss to what extent the benchmark model captures the structure of other models used in the literature, and whether this explains why other papers have found it difficult to match labor market flows. This review is by no means exhaustive, and we also discuss some papers that have no model, only tabulations of data. We convert monthly transition rates to continuous-time flows using the method of Shimer (2012), which also adjusts for time aggregation,[10] then calculate α^* and α_{data}, and the discrepancy

$$\Delta_\alpha = \alpha_{\text{data}} - \alpha^*$$

[10] Let P denote the monthly labor market transition probabilities. We find

$$Q = \begin{pmatrix} -(\lambda_{\text{EI}} + \lambda_{\text{EU}}) & \lambda_{\text{EU}} & \lambda_{\text{EI}} \\ \lambda_{\text{UE}} & -(\lambda_{\text{UE}} + \lambda_{\text{UI}}) & \lambda_{\text{UI}} \\ \lambda_{\text{IE}} & \lambda_{\text{IU}} & -(\lambda_{\text{IE}} + \lambda_{\text{IU}}) \end{pmatrix} \qquad \text{that satisfies} \qquad \exp(Q) = P$$

using the matrix logarithm (Higham 2008).

between the two values — when this is positive, the flows are outside the range of the benchmark model. Note since $\alpha \in [0,1]$, Δ_α is a unitless quantity that is easy to interpret. Table 2 summarizes the results, which we discuss in detail below.

	λ_{EU}	λ_{EI}	λ_{UE}	λ_{UI}	λ_{IU}	λ_{IE}	α_{data}	α^*	Δ_α
Andolfatto, Gomme, and Storer (1998)	.016	.016	.310	.155	.026	.022	.49	.83	.34
Fallick and Fleischman (2004)	.018	.026	.404	.330	.045	.035	.59	.88	.29
Garibaldi and Wasmer (2005) 15–64 yrs	.010	.016	.259	.166	.035	.044	.61	.80	.19
Garibaldi and Wasmer (2005) 25–54 yrs	.008	.010	.256	.133	.046	.034	.55	.71	.16
Pries and Rogerson (2009)	.011	.015	.234	.144	.038	.043	.58	.76	.19
Krusell et al. (2011) unadjusted	.018	.024	.385	.318	.041	.038	.57	.87	.30
Krusell et al. (2011) broad unemployment	.029	.016	.343	.336	.029	.064	.36	.80	.43

Table 2: Summary of various calibrations. Observed transition rates are monthly, corrected for time aggregation when necessary, displayed with 3 significant digits (calculations of course use the unrounded values). The last three columns contain the corresponding α_{data}, α^*, and Δ_α displayed with 2 significant digits. 0s before the decimal dot are omitted in order to obtain a compact table. Note that $\Delta_\alpha > 0$ for all papers indicating that the benchmark model cannot fit the data.

Andolfatto, Gomme, and Storer (1998) were among the first to emphasize the importance of the participation margin for modeling labor markets. Similarly to this paper they use a frictional labor market model that allows job offers for inactive workers with a probability that is lower compared to unemployed workers who search actively. They use $(w,v) \in \mathcal{X} = \mathbb{R}^2_+$ as a state for the workers where potential w is the wage and v is the potential value of home production. This formulation has the consequence that the unemployed in their model are those who have a have drawn a low wage and home production because if either one is larger than the other the worker will search actively or remain inactive. Also in their model the rate at which changes arrive to w is endogenous, because search will increase the probability of new offers and unemployment benefits are history-dependent. Despite these differences their model is very similar to our benchmark model, so it is not surprising that they cannot match labor market flows: the Δ_α calculated for their data is 0.34. They argue that the model has difficulties matching flows into and out of the labor force, but we have seen in Section 2 that this is not necessarily the case in this model family; however this view has influenced the subsequent literature.

The paper of Fallick and Fleischman (2004) contains no model, but they provide a detailed and methodologically thorough descriptive summary of gross labor market flows using CPS data between 1994:1–2003:12. We find that the discrepancy between α^* and α_{data} is $\Delta_\alpha = 0.29$.

Garibaldi and Wasmer (2005) present a model that is very close to the one in this paper — in fact the worker side of their baseline model is nested by our benchmark model in this paper, but their model is general equilibrium one, and is thus closed by modeling job creation. They use CPS data between 1995:10–2001:12 and calculate transition rates using the Abowd and Zellner (1985) correction. They argue that EI and IE flows are the result of time aggregation and misclassification, but allow for a positive job finding rate for the inactive ("jobs bump in to people") similarly to our model in their extended model. For their dataset, Δ_α is 0.16 (ages 25–54) and 0.19 (ages 16-64) for their dataset, which is the lowest among the papers we examine, and the results in Section 2 explain why they cannot calibrate to all six flows. Also, they target the share of marginally attached workers, which makes it even more difficult to calibrate to the data: in the benchmark model, this corresponds to ν, and as we lower q_ℓ to 0 to make α small, ν necessarily approaches 1, while in the data this is around 2% of the total population. Consequently, the flows from unemployment to inactivity they obtain from the model fall short of the data by an order of magnitude.

Pries and Rogerson (2009) use model similar to our benchmark model to motivate an explanation for cross-country differences in participation patterns. The most important difference between their model and the one in this paper is that theirs has a job-specific state and thus it can potentially provide richer flow patterns: we examine this possibility with a reduced form model in Section 4.2. Our model nests all other components of theirs as both feature linear utility and binary search decisions, and their only individual-specific state is a scalar that represents the cost associated with labor force participation and can take two values in their parameterization, and thus $\mathcal{X} = \{x_b, x_g\}$. They use March CPS data between 1990–2000, restricting ages between 16–64 years which yields $\Delta_\alpha = 0.19$. Consequently their model cannot match labor market flows, but following Andolfatto, Gomme, and Storer (1998) they also emphasize the model's inability to explain the magnitude of IU and UI flows.

Krusell et al. (2011) construct a three-state model with asset accumulation and nonlinear utility arguing that linear utility imposes implicit assumptions on income and substitution effects, which would prevent the discussion of the role of savings. In their model workers have a scalar productivity state s_t which evolves stochastically following an AR(1) process which is later extended with temporary shocks. Saving and consumption decisions also play a role in labor market transitions, but these differences turn out to have limited importance in practice—Section 6 of their paper discusses a setup with complete markets which is effectively similar to linear utility. The most important difference is that they allow only a single search intensity, arguing based on time-use surveys that search costs are small. In order to account for IE transitions they adjust transition rates by extending the notion of unemployment to include marginally attached workers. Calculation of Δ_α for both the unadjusted CPS data ($\alpha_{\text{data}} = 0.30$) and the flows with the extended unemployment state ($\alpha_{\text{data}} = 0.49$) suggest that this data adjustment makes it even more difficult to bring the model close to the data, which is apparent in their Table 6 which shows that the model cannot match α_{data} by a large margin. The most important reason for this is that λ_{UI} is relatively high to λ_{IU}, especially after adjusting the data.

In summary even though the papers discussed above use various modeling approaches and datasets (though mostly variants of the CPS), they cannot match labor market transition rates in the data. While formally the benchmark model presented in Section 2 only nests special parameterizations of some of these models, the corresponding α_{data}s combined with Lemma 1 suggests an explanation for this discrepancy. Following Andolfatto, Gomme, and Storer (1998), many of these papers talk about the difficulty of matching IU and UI flows, which is another way to interpret the results of Lemma 1: lowering λ_{UI} and increasing λ_{IU} flows would decrease the lower bound α^*.

Finally, since both the observed EI and EU transition rates are relatively small, it is reasonable to assess whether the mismatch between the benchmark model and the data could be a result of small sample sizes. In order to check this, we estimate transition rates using a Bayesian model, and draw posterior samples for both α^* and α_{data}. Since $\alpha^* - \alpha_{\text{data}}$ is the smallest for the flows in Garibaldi and Wasmer (2005) for ages 25-54, we use this dataset for the exercise.

Using a standard non-informative Dirichlet prior (Gelman et al. 2014, p 69), the sufficient statistics of the sample are the transition rates one obtains as point estimates from a simple tabulation and the sample size, which is inversely related to the precision of the posterior results. For illustration, we use a sample size of $N = 10000$, which is orders of magnitudes smaller than spanned by a decade of CPS data, which was used to obtain the tabulation.[11] We draw 10^4 points from the posterior, which allows us to summarize posterior probabilities with a very good precision.

The result is shown in Figure 3: for *all* 10^4 posterior draws,

$$\alpha^*(\text{benchmark model}) > \alpha_{\text{data}}$$

which means that the result is extremely unlikely to be an artifact of the sample size.

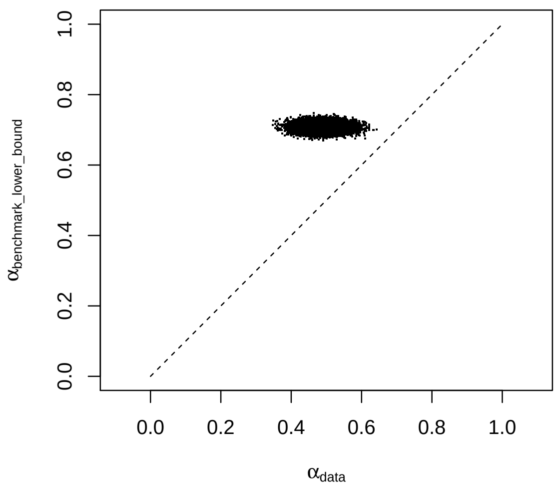

Figure 3: Posterior α_{data} vs the lower bound α^* of the benchmark model, with hypothetical sample size $N = 10000$, 10^5 posterior draws. The 45° line is dashed.

4 Extensions

Considering that $\alpha^* > \alpha_{\text{data}}$ for all the datasets reviewed in Section 3, we conclude that not only does the benchmark model discussed in Section 2 have a limited range of labor market flows it can generate, but the data appears to lie outside this range and thus the problem is empirically relevant.

In this section we discuss various extensions and check if they alleviate this problem. First, in Section 4.1 we introduce a continuous search effort margin, while in Section 4.2 we explore how state-dependent separation rates affect labor market flows and whether this alleviates the problem—as we

[11] This increases posterior uncertainty relative to the data: we do this to demonstrate that the results would be robust in an even smaller sample.

shall see, neither of these lower α^*. In Sections 4.3 and 4.4, we relax the assumption of having the same process for the individual-specific state x for all workers: since this plays a key role in the derivation of Lemma 1, it is not surprising that both extensions can make $\alpha^* < \alpha_{\text{data}}$. We aim for global analytical results in all of these discussions.

Finally, in Section 4.5 we consider the problem of measurement error in the form of survey misclassification: first, we provide a general theoretical characterization, then specifically examine the possibility of misclassification between inactivity and unemployment. We find that this is also helpful in matching the model to the data.

4.1 Continuous search effort

The binary search technology in the benchmark model is tractable, but very stylized. Time use studies such as Krueger and Mueller (2012) and Aguiar, Hurst, and Karabarbounis (2013) show that time devoted to searching for a job displays significant variation across countries, gender, and age, and thus it can be argued that a model with continuous search effort would be more realistic. This section extends the benchmark model with a search effort margin, and shows that when it comes to observed flows, the extended model can be mapped to the benchmark model and thus has the same constraints when it comes to matching observed transition rates between E, U, and I.[12]

The only change we make to the benchmark model is allowing the non-employed worker to choose the rate φ at which offers arrive continuously, by paying a search cost $c(\varphi)$. As is standard, we assume that c is continuous, nonnegative, strictly increasing, and convex.[13] The HJB equations are

$$rN(x) = b(x) + \max_{\varphi\geq 0} \overbrace{\left\{\varphi\big(W(x) - N(x)\big)^+ - c(\varphi)\right\}}^{\text{get offers at rate } \varphi\text{, pay search cost } c(\varphi)} + \gamma \overbrace{\Big(\mathbf{E}_{x'}\left[N(x')\right] - N(x)\Big)}^{\text{state change}} \tag{30}$$

$$rW(x) = w(x) + \underbrace{\sigma(N(x) - W(x))}_{\text{exogenous separation}} + \underbrace{\gamma(\mathbf{E}_{x'}[N(x') \vee W(x')] - W(x))}_{\text{state change, possibly endogenous separation}} \tag{31}$$

As before, let $S(x) = W(x) - N(x)$ and $s(x) = w(x) - b(x)$ denote the surplus value and the flow surplus. Then we can rewrite (30) and (31) as

$$(r+\sigma)S(x) = s(x) - \max_{\varphi\geq 0}\Big\{\varphi S(x)^+ - c(\varphi)\Big\} + \gamma\Big(\mathbf{E}_{x'}\left[S(x')^+\right] - S(x)\Big)$$

The second term on the right hand side is the opportunity cost of not searching, and the third term is the change of value from drawing a new x'. Introduce

$$\widehat{\varphi}(x) = \operatorname*{argmax}_{\varphi\geq 0} \varphi S(x)^+ - c(\varphi)$$

to denote the optimal search effort. From the assumptions on c, we know that $\widehat{\varphi}(x) = 0$ when $S(x) \leq 0$, and $\widehat{\varphi}$ is increasing in x. Assume that above some search effort $\overline{\varphi} > 0$, non-employed workers are

[12] I thank Fabien Postel-Vinay for suggesting this extension.

[13] See, for example, Christensen et al. (2005).

classified as *unemployed*, whereas for $\widehat{\varphi}(x) < \overline{\varphi}$ workers are classified as *inactive*. Let

$$\begin{aligned}
\mathrm{L} &= \{x : S(x) < 0\} \\
\mathrm{M} &= \{x : 0 \leq S(x), \widehat{\varphi}(x) < \overline{\varphi}\} \\
\mathrm{H} &= \{x : \overline{\varphi} \leq \widehat{\varphi}(x)\}
\end{aligned}$$

Then define the observed job finding rates for workers in M and H as

$$\varphi_{\mathrm{m}} \equiv \int_{x \in \mathrm{M}} \widehat{\varphi}(x) dF(x) \tag{32}$$

$$\varphi_{\mathrm{h}} \equiv \int_{x \in \mathrm{H}} \widehat{\varphi}(x) dF(x) \tag{33}$$

Since all other parts of the benchmark model are unchanged, it is easy to see that with (32) and (33), observationally this model can be mapped to the benchmark model. Consequently, all the conclusion about the benchmark model apply, in particular, the lower bound α^* is the same as in the benchmark model, and thus this extension does not resolve the discrepancy between the model and the data.

4.2 State-dependent separation rates

In this section, we relax the assumption that exogenous separation rates are the same for all employed workers. We can rationalize this as a reduced-form version of a model in which some jobs are less stable than others: it would not be unreasonable to assume that jobs in which the workers's surplus is lower are less able to withstand certain kinds of exogenous shocks.[14]

For analytical simplicity we only distinguish separation rates for marginal and non-marginal workers: non-marginal workers separate at rate σ, while marginal workers separate at a higher rate $\sigma + \delta_\sigma$, where $\delta_\sigma \geq 0$. Everything else is the same as in the benchmark model: in particular, the only observed flow that is different compared to the benchmark model is

$$\lambda_{\mathrm{EI}} = \mu(\sigma + \delta_\sigma) + q_\ell \tag{34}$$

Consequently,

$$\lambda_{\mathrm{E[IU]}} = \sigma + \mu\delta_\sigma + q_\ell \tag{35}$$

The flow balance equations (11)–(14) are unchanged, but for e_{m} and e_{h} we have

$$\begin{aligned}
e_{\mathrm{m}} \cdot (q_\ell + q_{\mathrm{h}} + \delta_\sigma + \sigma) &= e_{\mathrm{h}} \cdot q_{\mathrm{m}} + (m_{\mathrm{m}} - e_{\mathrm{m}}) \cdot \varphi_{\mathrm{m}} \\
e_{\mathrm{h}} \cdot (q_{\mathrm{m}} + q_\ell + \sigma) &= e_{\mathrm{m}} \cdot q_{\mathrm{h}} + (m_{\mathrm{h}} - e_{\mathrm{h}}) \cdot \varphi_{\mathrm{h}}
\end{aligned}$$

Similarly to Section 2, we first derive simpler bounds for α: leaving in q_ℓ and δ_σ as free parameters and

[14] I thank Christian Haefke for suggesting this extension.

using (7), (8), (10), and (35) we find that there is a lower bound on $\underline{\mu} \leq \mu(q_\ell, \delta_\sigma)$, defined implicitly by

$$\underline{\mu} = \frac{q_\mathrm{m}}{q_\mathrm{m} + q_\ell + q_\mathrm{h} + \delta_\sigma + \sigma} = \frac{\lambda_\mathrm{UI} - q_\ell}{\lambda_\mathrm{UI} + \lambda_\mathrm{IU} + \lambda_{\mathrm{E[IU]}} + (1 - \underline{\mu})\delta_\sigma - q_\ell} \tag{36}$$

and from (34),

$$\alpha = \frac{(1 - \mu)\mu\delta_\sigma + \mu\lambda_{\mathrm{E[IU]}} + (1 - \mu)q_\ell}{\lambda_{\mathrm{E[IU]}}} \tag{37}$$

However, we can't just plug (36) into (37), since the former has μ on the right hand side. Solving for μ explicitly is cumbersome, since (36) is quadratic in μ. However, there is a simple transformation that helps: define

$$z_\sigma = \underline{\mu} \cdot (1 - \underline{\mu}) \cdot \delta_\sigma$$

and then transform (36) and (37) into

$$\begin{aligned} \underline{\mu}(q_\ell, z_\sigma) &= \frac{z_\sigma - \lambda_\mathrm{UI} + q_\ell}{q_\ell - \lambda_\mathrm{UI} - \lambda_\mathrm{IU} - \lambda_{\mathrm{E[IU]}}} \\ \alpha \geq \underline{\alpha}(q_\ell, z_\sigma) &= \frac{\underline{\mu}\lambda_{\mathrm{E[IU]}} + (1 - \underline{\mu})q_\ell + z_\sigma}{\lambda_{\mathrm{E[IU]}}} \end{aligned}$$

When $\delta_\sigma = 0$, then $z_\sigma = 0$, and z_σ is weakly increasing in δ. Combining the two equations above,

$$\alpha \geq \underline{\alpha}(q_\ell, z_\sigma) = \frac{z_\sigma(\lambda_\mathrm{UI} + \lambda_\mathrm{IU}) + \lambda_{\mathrm{E[IU]}}\lambda_\mathrm{UI} + \lambda_\mathrm{IU} q_\ell}{\lambda_{\mathrm{E[IU]}}(\lambda_\mathrm{UI} + \lambda_\mathrm{IU} + \lambda_{\mathrm{E[IU]}} - qL)} \tag{38}$$

Note that the right hand side of (38) is increasing in z, and thus it is lowest when $z = 0$, which happens when $\delta_\sigma = 0$.[15] Also, when $z_\sigma = 0$ and $q_\ell = 0$, (38) is equivalent to (28), the simple bound for the benchmark model. The lemma below shows this result for the exact bound.

Lemma 2 (Bounds for $\alpha(q_\ell, \delta_\sigma)$ in the state dependent separation rate model)**.** *When the state dependent separation rate model is calibrated to* λ_IU*,* λ_UI*,* λ_IE*,* λ_UE*, and* $\lambda_{\mathrm{E[IU]}}$*, with* $q_\ell \geq 0$ *and* $\delta_\sigma \geq 0$ *as free parameters,*

1. $\alpha(q_\ell, \delta_\sigma)$ *is strictly increasing in* q_ℓ *and* δ_σ*,*
2. *and has the same lower bound as in the benchmark model, characterized by Lemma 1.*

Proof. The proof is in the online Appendix A, and combines the logic of the proof of Lemma 1 with the transformation $z_\sigma = \mu(1 - \mu)\delta_\sigma$. □

The intuition for why α is increasing in δ_σ is the same as for q_ℓ: increasing δ_σ increases the share of EI flows for a given μ (direct effect) because marginal employed workers separate into inactivity, but δ_σ itself decreases μ since fewer workers remain in this state. Overall, the direct effect also dominates.[16] We conclude that with this extension we do not improve the ability of the benchmark model to match the data.

[15] Except for the special cases $\mu = 0$ or $\mu = 1$, but this does not change the conclusion.

[16] Interestingly, z is always increasing in δ_σ, and α and μ are both increasing in δ_σ, so with this transformation we don't even need to prove that the direct effect is stronger.

4.3 Permanently inactive population

A crucial assumption that underlies the benchmark model is that the process for the individual-specific state x is independent of employment status, specifically that

1. the arrival rate γ of change events is the same for employed and non-employed workers,
2. conditional on a change event, employed and non-employed workers draw their next state x' from the same IID distribution F.

In this section we relax this assumption for inactive workers, while in Section 4.4 we introduce different distributions for the employed and non-employed. The IU transition rate is $\lambda_{\text{IU}} \approx 0.05$, which means that on average, an inactive worker would join the labor force within 1.5–2 years. However, it is reasonable to assume that for some workers, withdrawal from the labor force is a temporary phenomenon that lasts for a short duration, while some other workers, entry into the labor force would be unlikely or impossible, for example because of a permanent disability.

We model this in a very stylized manner by assuming that a fraction ζ of the inactive population is *permanently inactive*: these workers remain in the region L, never experiencing any transitions to x (ie for them, $\gamma = 0$), and consequently would never accept a job and don't contribute to any of the observed gross flows. In contrast, the rest of the workers experience shocks to their state x according to the benchmark model of Section 2, and would never become permanently inactive — in other words, there is no mixing between the two groups. While assuming that workers could enter and leave the state of being permanently inactive, perhaps with a low transition rate, might be more realistic, this assumption simplifies the exposition considerably: we can simply write observed flows as a mixture of the flows for the benchmark model and 0, weighted by the relevant share of permanently inactive workers.

Specifically, let $\lambda'_{\text{IU}}, \lambda'_{\text{IE}}, \dots$ denote the transition rates of the benchmark model, and $\lambda_{\text{IU}}, \lambda_{\text{IE}}, \dots$ the observed transition rates as before. Since the share of permanently inactive workers among I is ζ, we have

$$\lambda_{\text{IU}} = \zeta \cdot 0 + (1-\zeta) \cdot \lambda'_{\text{IU}} = (1-\zeta)\lambda'_{\text{IU}} \tag{39}$$

and

$$\lambda_{\text{IE}} = \zeta \cdot 0 + (1-\zeta) \cdot \lambda'_{\text{IE}} = (1-\zeta)\lambda'_{\text{IE}}$$

Moreover, since the share of permanently inactive workers among the unemployed and employed is zero, the other transition rates are unaffected:

$$\lambda_{\text{UI}} = \lambda'_{\text{UI}}, \qquad \lambda_{\text{UE}} = \lambda'_{\text{UE}}, \qquad \lambda_{\text{EI}} = \lambda'_{\text{EI}}, \qquad \lambda_{\text{EU}} = \lambda'_{\text{EU}}.$$

Effectively, this extension of the model can be seen as nothing more than a transformation of the data, which imposes

$$\lambda'_{\text{IE}} = \frac{\lambda_{\text{IE}}}{1-\zeta} \qquad \lambda'_{\text{IU}} = \frac{\lambda_{\text{IU}}}{1-\zeta}$$

Using Lemma 1, it is easy to show the following result.

Lemma 3 (Permanently inactive population.). *Let $\alpha^*(\zeta)$ denote the lowest α that the model can generate while matching the flows λ_{IE}, λ_{IU}, λ_{UI}, λ_{UE}, and $\lambda_{\text{E[IU]}}$, extending the benchmark model with ζ permanently inactive workers a share of all inactive workers, with $q_\ell = 0$. Then*

$$\frac{\partial}{\partial \zeta}\alpha^*(\zeta) < 0 \qquad \textit{and} \qquad \lim_{\zeta \to 1} \alpha^*(\zeta) = 0$$

Proof. Substituting (29) and (39) into (29) *mutatis mutandis*, then the results follow by differentiation and taking the limit. □

The intuition is the following: in Section 2.4 we have seen that one of the things that keep α high in the model is thatλ_{IU} is small relative to λ_{UI}. A higher ζ concentrates the same number of observed transitions among fewer workers, effectively increasing λ_{IU}, and thus lowering α^* because fewer of the employed workers will be marginal, since most of them come from unemployment.

In the limiting case where ζ approaches 1, α can be driven arbitrarily low, so in a purely mechanical sense this solves the problem faced by the benchmark model. However, for practical purposes it is interesting to see what ζ is required to align the data with the model, ie

$$\zeta^* \equiv \min\{\zeta : \alpha(\zeta) = \alpha_{\text{data}}\}.$$

Table 3 shows ζ^* for various datasets. Clearly, a large fraction of the inactive workers need to be outside the labor market for this extension of the model to match the data: the lowest value of ζ^* is 55%, but values between 80%–90% are more common. Deciding whether this fraction is plausible requires more

	ζ^*
Andolfatto, Gomme, and Storer (1998)	0.86
Fallick and Fleischman (2004)	0.84
Garibaldi and Wasmer (2005) 15–64 yrs	0.66
Garibaldi and Wasmer (2005) 25–54 yrs	0.55
Pries and Rogerson (2009)	0.63
Krusell et al. (2011) unadjusted	0.84
Krusell et al. (2011) broad unemployment	0.89

Table 3: Minimum share of permanently inactive workers that matches the data to the model.

careful analysis, in particular of long-term transitions of initially inactive workers. However, while the very stylized model presented in this section demonstrates a theoretical point very simply, it may be difficult to map to the data, as the distinction between workers with various levels of attachment to the labor force may not be so stark as implied by this model, and the data may require a richer structure and occasional transitions between long-term inactive workers and the rest of the population. We leave this for future research.

4.4 Learning on the job

Intuitively, the ratio α is constrained in the benchmark model of Section 2 because we impose the same process for the individual-specific state x regardless of employment status. Once we lift this restriction, there are many ways to generalize the model, depending on what kind of difference we would like to emphasize between employment and non-employment.

In general, these generalizations can break the connection between the flows of the employed and the non-employed that we used to derive Lemma 1. We illustrate this with an example where the benchmark model is extended with *learning on the job*: at rate δ_{h}, employed workers experience an exogenous shock which puts their state x in the region H where they have a high surplus. Intuitively, we think of this as a very stylized model of skill gain for the employed, but note that since δ_{h} simply drives a wedge between transition probabilities for the employed and the non-employed, we can also think of this as a stylized model of skill loss for the non-employed, similarly to Ljungqvist and Sargent (1998), Ljungqvist and Sargent (2004), Haan, Haefke, and Ramey (2005), Ljungqvist and Sargent (2007b), and Ljungqvist and Sargent (2008).

In contrast to the benchmark model, because the process for x is not independent of the employment status, we cannot write the flow balance equations for the totals M and H independently of e_{m} and e_{h}. Only (11) is unchanged, and for employment e_{m} and e_{h} and non-employment $m_{\mathrm{m}} - e_{\mathrm{m}}$ and $m_{\mathrm{h}} - e_{\mathrm{h}}$ we have the flow balance equations

$$
\begin{aligned}
(m_{\mathrm{m}} - e_{\mathrm{m}}) \cdot (\varphi_{\mathrm{m}} + q_\ell + q_{\mathrm{h}}) &= (m_\ell + m_{\mathrm{h}} - e_{\mathrm{h}}) \cdot q_{\mathrm{m}} + e_{\mathrm{m}} \cdot \sigma \\
(m_{\mathrm{h}} - e_{\mathrm{h}}) \cdot (\varphi_{\mathrm{h}} + q_\ell + q_{\mathrm{m}}) &= (m_\ell + m_{\mathrm{m}} - e_{\mathrm{m}}) \cdot q_{\mathrm{h}} + e_{\mathrm{h}} \cdot \sigma \\
e_{\mathrm{m}} \cdot (q_\ell + q_{\mathrm{h}} + \boxed{\delta_{\mathrm{h}}} + \sigma) &= e_{\mathrm{h}} \cdot q_{\mathrm{m}} + (m_{\mathrm{m}} - e_{\mathrm{m}}) \cdot \varphi_{\mathrm{m}} \\
e_{\mathrm{h}} \cdot (q_\ell + q_{\mathrm{m}} + \sigma) &= e_{\mathrm{m}} \cdot (q_{\mathrm{h}} + \boxed{\delta_{\mathrm{h}}}) + (mH - eH) \cdot \varphi_{\mathrm{h}}
\end{aligned}
$$

The interpretation of the above equations is similar to Section 2.3, and the only difference is the highlighted term δ_{h}: this causes an outflow $e_{\mathrm{m}}\delta_{\mathrm{h}}$ from the marginal employed, which appears as an inflow to the non-marginal employed. The characterization of μ, ν and α can be done in a manner similar to the benchmark model, which leads to the following lemma.

Lemma 4 (Bounds for $\alpha(q_\ell, \delta_{\mathrm{h}})$ in the model with learning on the job)**.** *When the model with learning on the job is calibrated to λ_{IU}, λ_{UI}, λ_{IE}, λ_{UE}, and $\lambda_{\mathrm{E[IU]}}$, with $q_\ell \geq 0$ and $\delta_{\mathrm{h}} \geq 0$ as free parameters*

1. *$\alpha(q_\ell, \delta_{\mathrm{h}})$ is strictly increasing in q_ℓ,*

2. *for a given δ_{h}, $\alpha(q_\ell, \delta_{\mathrm{h}})$ has the lower bound*

$$
\alpha^* = \alpha(0, \delta_{\mathrm{h}}) = \frac{\lambda_{\mathrm{UI}}}{\lambda_{\mathrm{UI}} + \lambda_{\mathrm{IU}} + \lambda_{\mathrm{E[IU]}}(1 - \Delta) + \delta_{\mathrm{h}}} \tag{40}
$$

where

$$
\Delta = \frac{\lambda_{\mathrm{IE}}(\lambda_{\mathrm{E[IU]}} + \lambda_{\mathrm{IU}} + \lambda_{\mathrm{UE}} + \delta_{\mathrm{h}}) + \lambda_{\mathrm{IE}}\lambda_{\mathrm{UI}}}{\lambda_{\mathrm{IE}}(\lambda_{\mathrm{E[IU]}} + \lambda_{\mathrm{UE}} + \lambda_{\mathrm{UI}}) + \lambda_{\mathrm{IU}}\lambda_{\mathrm{UE}}}
$$

Proof. The proof is in the online Appendix A. □

This implies that by increasing δ_{h}, we can drive α arbitrarily low.

Corollary 1. *For this model, $\alpha(0, \delta_{\mathrm{h}})$ is decreasing in δ_{h}, and in particular*

$$\lim_{\delta_{\mathrm{h}} \to \inf} \alpha(0, \delta_{\mathrm{h}}) = 0$$

Proof. Follows from differentiating (40) and taking limits. See Appendix A. □

The intuition behind these results is simple: shocks that result in "learning on the job" draw employed workers away from e_{m}, into e_{h}, and thus decrease μ. There is no direct effect on α, as δ_{h} works via the composition effect through μ only.

calibration ($q_\ell = 0$)	δ_{h}^*
Andolfatto, Gomme, and Storer (1998)	0.134
Fallick and Fleischman (2004)	0.189
Garibaldi and Wasmer (2005)	0.065
Garibaldi and Wasmer (2005) 25–54 yrs	0.057
Pries and Rogerson (2009)	0.064
Krusell et al. (2011) unadjusted	0.194
Krusell et al. (2011) broad unemployment	0.528

Table 4: Lowest rates for learning on the job (δ_{h}) that match the observed flows at $q_\ell = 0$, for various calibrations.

Similarly to Section 4.3, we calculate the lowest δ_{h} that would make $\alpha_{\text{data}} = \alpha(0, \delta_{\mathrm{h}})$, and thus have the model match the observed flows at $q_\ell = 0$. The results are shown in Table 4. A more intuitive way of interpreting them is $1/\delta_{\mathrm{h}}$, which is the expected time until a marginal employed worker gains the skills $x \in \mathrm{H}$ — this is between 5 and 15 months for most datasets, with the exception of the broad unemployment measure of Krusell et al. (2011). This may be because the discrepancy $\alpha^* - \alpha_{\text{data}}$ is largest for this dataset for the benchmark model for reasons we discussed in Section 3, and a large δ_{h} is needed to make up for the difference.

4.5 Misclassification

In this section we explore the implications of misclassification for labor market data. The seminal article of Flinn and Heckman (1983) shows that inactivity and unemployment a distinct states, but recent literature has shown that the line between the two is not very well defined. Sorrentino (1995) demonstrates that unemployment is very sensitive to different definitions, while Jones and Riddell (1999) argue that marginal inactives are closer in behavior to the unemployed than the rest of the inactives. Elsby, Hobijn, and Şahin (2015) suggest removing spurious flows using a simple algorithm that removes single-period outliers.

Potential misclassification between unemployment and inactivity is mostly relevant to the results in this paper because occasional misclassification of workers may result in spurious flows. Since all flows are related, we cannot just adjust them in an *ad hoc* manner, and in order to examine this issue consistently, we need to specify a data generating process that relates underlying, unobserved flows to observed flows via a misclassification error. In Section 4.5.1, we develop a framework for a very simple type of misclassification, in which misclassification errors are independent and identically distributed across periods, and show how to infer the underlying transition probabilities from observed ones. This formulation is general. We then apply it to a simple misclassification specification, in which unemployed workers are recorded as inactive, and vice versa, with given probabilities. Parameterizing the problem with these probabilities, we examine their effect on α^* and α_{data}.

4.5.1 Theoretical framework

Fix time period t, and let p_{ij} denote the probability of transitioning to j at $t+1$, conditional on the state being i at t. We assume that the states $i, j \in \mathcal{I}$ are not directly observed, but classified according to an IID multinomial distribution: at t and $t+1$, state i is observed as $k \in \mathcal{K}$ with probability m_{ik}, and we observe transition probabilities h_{ij}. We emphasize that $\mathcal{I}$ and $\mathcal{K}$ need not coincide in the derivation below, which is more general, even though for the specific problem we don't make use of this.

Let π_i denote distribution of states at t, and π the corresponding vector. Let P, M, and H denote the respective stochastic transition matrices, we call M the *misclassification matrix*. We now characterize H as a function of P and M. We emphasize that the space of hidden states (eg i, j) and the space of observed states (k) need not coincide or even have the same cardinality, so M may not be square. In the equation below, we always assume that i and j are summed over hidden states, while k and l are summed over observed states, without making this notation explicit.

A special case is no observational error: when $M = I$ (no classification errors), $H = P$. For the general case, introduce the conditional probability that having observed k, the underlying state is i, as

$$s_{ki}(M, \pi) = \Pr(\text{state at } t = i \mid \text{observing } k \text{ at } t) = \frac{m_{ik}\pi_i}{\sum_i m_{ik}\pi_i} \tag{41}$$

which follows from Bayes' rule. Then we can write the observed transition probabilities as

$$h_{kl}(P, M, \pi) = \sum_{i,j} s_{ki}(M, \pi) p_{ij} m_{jl}$$

which can be written in a more compact form as

$$H(P, M, \pi) = S(M, \pi) P M \tag{42}$$

Now we solve the inverse problem: given observed transition probabilities H and assuming a misclassification matrix M, we would like to infer P. Since S is a function of π and M, we also need to make some assumptions about π. For this exercise, we impose that π is the steady state distribution under P, ie

$$\pi = \pi P \tag{43}$$

This assumption is innocuous, since under reasonable conditions distributions converge to the steady state relatively quickly.[17]

Formally, given H and M, we are looking for a solution for P, π and S such that (41), (42) and (43) hold, and of course all probabilities are proper, $\sum_i \pi_i = 1, \sum_j p_{ij} = 1 \,\forall j, \pi_i \geq 0, p_{ij} \geq 0 \,\forall i,j$. Notice that if we knew π, then we could calculate S, and solve for P in (42). The following lemma shows that it is easy to find π: intuitively, if η is a steady state distribution under the transition matrix H, then $\pi M = \eta$.

Lemma 5 (Steady state distribution π from M and H.)**.** *Fix H and M. When π is a steady state distribution such that* (43) *holds, and for the resulting S* (41) *the equation* (42) *holds, then*

$$\pi M H = \pi M \tag{44}$$

or in other words, for

$$\eta = \pi M \tag{45}$$

we have $\eta H = \eta$.

Proof. For any element of H,

$$h_{kl} = \sum_j (SP)_{kj} m_{jl} = \sum_{i,j} s_{ki} p_{ij} m_{jl} = \sum_{i,j} \frac{m_{ik}\pi_i}{\sum_{\hat{\imath}} m_{\hat{\imath}k}\pi_{\hat{\imath}}} p_{ij} m_{jl}$$

where the first equality follows the definition of matrix multiplication, the second from (42), the third from (41). Multiply both sides by the sum in the denominator and sum by k to obtain

$$\sum_{\hat{\imath},k} \pi_{\hat{\imath}} m_{\hat{\imath}k} h_{kl} = \sum_{i,j,k} m_{ik}\pi_i p_{ij} m_{jl} = \sum_{i,j} \pi_i p_{ij} m_{jl} = \sum_j \pi_j m_{jl} \tag{46}$$

where the second equality follows from $\sum_k m_{ik} = 1$ and the third from (43). Equation (44) is equivalent to (46) in matrix notation. □

Then P is calculated as follows: we find the steady state distribution η of H, solve for π from (45) given M, then calculate S and solve (42) for P. It it important to note that the these calculations do not ensure that P is a proper stochastic transition matrix for arbitrary misclassification matrices M: in particular, large off-diagonal values for M may make elements in P negative. We discuss this below for the concrete application.

4.5.2 Misclassification between unemployment and inactivity

Now we apply the results of Section 4.5.1 to labor market flows. For the sake of simplicity, we assume that employment can be observed without any ambiguity, while the boundary between unemployment

[17] Following Shorrocks (1978, Section 3), we can calculate the half-life of the Markov process described by a matrix P as

$$h = -\frac{\log 2}{\log |\lambda_2|}$$

where $1 = \lambda_1 \geq |\lambda_2| \geq |\lambda_3| \geq \dots$ are the eigenvalues of P. For labor market flows, the half life is typically between 6–8 months, which means that (43) is a reasonable assumption.

and inactivity is less well-defined and thus workers in either category may end up misclassified into the other. Specifically, with an ordering of states as E, U, and I, we use a misclassification matrix

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 - r_{\text{UI}} & r_{\text{UI}} \\ 0 & r_{\text{IU}} & 1 - r_{\text{IU}} \end{pmatrix}$$

Then the conditional probability matrix for observations is

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 - s_{\text{UI}} & s_{\text{UI}} \\ 0 & s_{\text{IU}} & 1 - s_{\text{IU}} \end{pmatrix} \quad \text{with} \quad s_{\text{UI}} = \frac{r_{\text{UI}}\pi_{\text{U}}}{(1 - r_{\text{IU}})\pi_{\text{I}} + r_{\text{UI}}\pi_{\text{U}}}, s_{\text{IU}} = \frac{r_{\text{IU}}\pi_{\text{I}}}{r_{\text{IU}}\pi_{\text{I}} + (1 - r_{\text{UI}})\pi_{\text{U}}}$$

Taking observed transition H as given and $(r_{\text{IU}}, r_{\text{UI}})$ as parameters, we solve (45) for π, then obtain the underlying transition matrix P.[18] Given P, we find the continuous-time transition rates Λ,[19] then using Λ, we can analyze the discrepancy between the model and the data, in particular calculate α_{data} and the lower bound α^* for the benchmark model. The latter step is performed numerically, for but the analytical results below we explain the intuition for the effect of misclassification on the elements of P, as there is a monotone connection between off-diagonal elements of P and the corresponding continuous-time transition rates.

Since M and S are invertible, we can obtain closed-form solutions. First, we find that

$$p_{\text{EI}} = \frac{h_{\text{EI}} - r_{\text{UI}} (h_{\text{EU}} + h_{\text{EI}})}{1 - (r_{\text{IU}} + r_{\text{UI}})} \qquad p_{\text{EU}} = \frac{h_{\text{EU}} - r_{\text{IU}} (h_{\text{EU}} + h_{\text{EI}})}{1 - (r_{\text{IU}} + r_{\text{UI}})}$$

These equations are very intuitive. If there is no misclassification ($r_{\text{IU}} = 0, r_{\text{UI}} = 0$), $h_{\text{EI}} = p_{\text{EI}}$ and $h_{\text{EU}} = p_{\text{EU}}$. Also, $p_{\text{EI}} + p_{\text{EU}} = h_{\text{EI}} + h_{\text{EU}}$ regardless of r_{IU} and r_{UI}, because the misclassification we consider moves individuals between the I and U states, and the total outflow from E is unaffected.

For the other four parameters, the closed form solution is complicated without a significant gain in insight, so we use perturbation methods for illustration.[20] We obtain the following linear approximation by implicit differentiation around $r_{\text{IU}} = 0$, $r_{\text{UI}} = 0$:

$$\begin{pmatrix} p_{\text{IU}} \\ p_{\text{IE}} \\ p_{\text{UI}} \\ p_{\text{UE}} \end{pmatrix} \approx \begin{pmatrix} h_{\text{IU}} \\ h_{\text{IE}} \\ h_{\text{UI}} \\ h_{\text{UE}} \end{pmatrix} + r_{\text{IU}} \begin{pmatrix} -\frac{1}{C}(h_{\text{UU}} - h_{\text{IU}}) - h_{\text{II}} \\ -\frac{1}{C}(h_{\text{UE}} - h_{\text{IE}}) \\ h_{\text{UI}} \\ 0 \end{pmatrix} + r_{\text{UI}} \begin{pmatrix} h_{\text{IU}} \\ 0 \\ -C(h_{\text{II}} - h_{\text{UI}}) - h_{\text{UU}} \\ C(h_{\text{UE}} - h_{\text{IE}}) \end{pmatrix} \tag{47}$$

with the constants

$$C = \frac{h_{\text{EU}}h_{\text{IE}} + h_{\text{EI}}h_{\text{IU}} + h_{\text{EU}}h_{\text{IU}}}{h_{\text{EU}}h_{\text{UI}} + h_{\text{EI}}h_{\text{UI}} + h_{\text{EI}}h_{\text{UE}}} \tag{48}$$

In the data, the job finding probability of unemployed is larger than the job finding probability of the

[18] As usual, algebraic details are relegated to Appendix A.

[19] See footnote 10.

[20] The numerical calculations for Figure 4 are exact.

inactive, hence

$$h_{\mathrm{UE}} > h_{\mathrm{IE}} \tag{49}$$

For monthly and quarterly frequencies, the probabilities of staying unemployed or inactive are large, so we can assume

$$h_{\mathrm{IU}} < h_{\mathrm{UU}} \qquad \text{and} \qquad h_{\mathrm{UI}} < h_{\mathrm{II}} \tag{50}$$

The table below summarizes the signs of the coefficients of r_{IU} and r_{UI} in (47), given (49) and (50).

vs.	r_{IU}	r_{UI}
p_{IU}	$-$	$+$
p_{IE}	$-$	0
p_{UI}	$+$	$-$
p_{UE}	0	$+$

Similar to Section 2, we use monthly data from Garibaldi and Wasmer (2005).[21] First, note that because

$$h_{\mathrm{UI}} \gg h_{\mathrm{IU}} \qquad \text{and} \qquad h_{\mathrm{EI}} h_{\mathrm{UE}} \gg h_{\mathrm{EU}} h_{\mathrm{IE}},$$

the numerator in (48) is much smaller than the denominator, which makes

$$C \approx 0.22$$

Consequently,

$$\begin{pmatrix} p_{\mathrm{IU}} \\ p_{\mathrm{IE}} \\ p_{\mathrm{UI}} \\ p_{\mathrm{UE}} \\ p_{\mathrm{EI}} \\ p_{\mathrm{EU}} \end{pmatrix} \approx \begin{pmatrix} 0.036 \\ 0.037 \\ 0.106 \\ 0.212 \\ 0.010 \\ 0.007 \end{pmatrix} + r_{\mathrm{IU}} \cdot \begin{pmatrix} -3.796 \\ -0.780 \\ 0.106 \\ 0.000 \\ 0.010 \\ -0.010 \end{pmatrix} + r_{\mathrm{UI}} \cdot \begin{pmatrix} 0.036 \\ 0.000 \\ -0.864 \\ 0.039 \\ -0.007 \\ 0.007 \end{pmatrix}$$

The larger r_{IU} is, the more inactive workers are misclassified as unemployed. The primary effect of this is decreasing IU transition probabilities p_{IU}, since $\pi_I \gg \pi_U$, but it also decreases p_{IE} and has a small increasing effect on p_{UI}. Since UI flows are actually increased relative to IU, it is easy to see from (29) that increasing r_{IU} will increase α^*. In a certain sense, r_{UI} has the opposite effect as r_{IU}: introducing misclassification from U to I increases p_{IU} and decreases p_{UI}, which is just what we need to decrease α^*.

It is important to note that not all $r_{\mathrm{UI}}, r_{\mathrm{IU}} \in [0, 1]$ values are admissible for a given problem, since values that are too large lead to negative steady state and transition probabilities. For example, consider the mapping

$$\pi = M^{-1}\eta \qquad \Rightarrow \qquad \pi_{\mathrm{E}} = \eta_{\mathrm{E}}, \quad \pi_{\mathrm{U}} = \frac{\eta_{\mathrm{U}} - (\eta_{\mathrm{U}} + \eta_{\mathrm{I}}) r_{\mathrm{IU}}}{1 - (r_{\mathrm{UI}} + r_{\mathrm{IU}})}, \quad \pi_{\mathrm{I}} = \frac{\eta_{\mathrm{I}} - (\eta_{\mathrm{U}} + \eta_{\mathrm{I}}) r_{\mathrm{UI}}}{1 - (r_{\mathrm{UI}} + r_{\mathrm{IU}})}$$

[21] CPS, ages 25–54.

which would require that

$$r_{\mathrm{IU}} \leq \frac{\eta_{\mathrm{U}}}{\eta_{\mathrm{U}} + \eta_{\mathrm{I}}}, \quad r_{\mathrm{UI}} \leq \frac{\eta_{\mathrm{I}}}{\eta_{\mathrm{U}} + \eta_{\mathrm{I}}},$$

Intuitively, when, for example, r_{IU} is higher than this, the implied mass of observed unemployed would exceed the actual steady state value. This is especially constraining for r_{IU}, since $\eta_{\mathrm{U}} \ll \eta_{\mathrm{I}}$. Because solving (42) for P also requires inverting S^{-1}, we have even more constraints of the same kind, however they are not shown explicitly here because they are not very intuitive — we simply check that $P \geq 0$ and $\pi \geq 0$ in the numerical calculations below, and only show the admissible range on the graphs.

Motivated by this, we explore the effect of r_{UI} on the lower bound α^* of (29) using exact calculations instead of linear approximations: setting $r_{\mathrm{IU}} = 0$, for a given r_{UI} we calculate P from H, then transform to Λ, and use (1) and (29). Figure 4 shows $\alpha_{\text{data}} = \lambda_{\mathrm{EI}}/\lambda_{\mathrm{E[IU]}}$ and the lower bound α^* as a function of r_{UI}. Notice that a misclassification probability of around 9% can align the model with the data. Also note, however, that the numbers allow a maximum rate of 14% for the misclassification probability r_{UI}, as higher values than that this would imply a negative unemployment stock. We conclude that misclassification is a promising solution for solving the puzzle posed by Section 2, but further work is needed to investigate its plausibility.

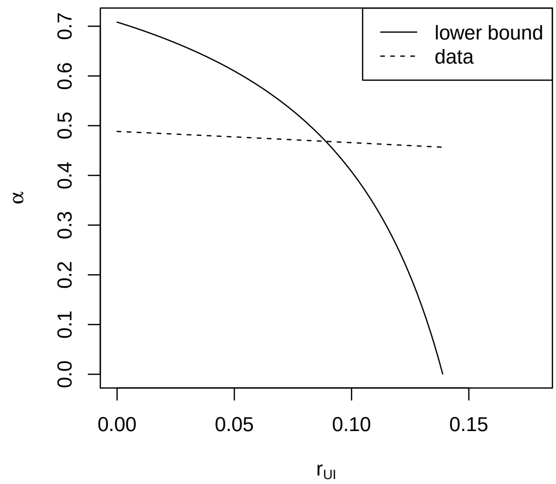

Figure 4: α as a function of the UI misclassification probability r_{UI} and its lower bound in the benchmark model.

5 Conclusion

We have provided a tractable analytical characterization of the range of labor market transition rates that can be generated by a benchmark model that nests or is similar to commonly used models in the literature. This model cannot match labor market transition rates observed in the data, and the main reason for this discrepancy is that UI transition rates are large relative to IU, EI and EU transition rates while at the same time EI flows are a relatively small share of all flows out of employment. We would like to emphasize that even though relative magnitudes of some flows help us think about this problem intuitively, the failure of the model to match the data is not attributable to the magnitude of a single flow.

Models with a similar logic as in Section 2 are commonly used in the literature, and thus it is important to be aware of the fact that when it comes to replicating gross labor market flows, there is a missing ingredient. Having considered various extensions, the following modifications to the model could help to bring it closer to the data:

1. Inactive workers who rarely experience labor market transitions: in the stylized model we discussed in Section 4.3, they form a completely separate group from the rest of the population, but in a more realistic model it would be reasonable to assume some mixing.

2. Different productivity processes for the employed and the nonemployed. Even though the model of Section 4.4 motivated the process as learning on the job, all we need is a wedge between productivity processes for the employed and the non-employed.

3. Misclassification of labor market states, particularly occasionally observing the unemployed as inactive. We have only scratched the surface with the particular specification in Section 4.5, and much richer models are conceivable, including non-IID misclassification where the probability of being observed as unemployed would be increasing in the surplus, or misclassification for only the marginal inactives.

Since these extensions to the benchmark model are all capable of matching monthly gross flows between employment, unemployment and inactivity, it is clear that they cannot be used to identify which model is more relevant empirically. The next logical step would be checking the fit of the model to labor market flow patterns across multiple periods, which are possible to obtain from both the CPS and the LFS. Also, both Garibaldi and Wasmer (2005) and Krusell et al. (2011) show that it is important to account for marginally attached workers, so they should perhaps be considered a different state than inactivity. Also, the misclassification process could be made richer this way: for example, it is plausible to argue that while unemployment and non-marginal inactivity are relatively well-defined, marginally attached workers are more likely to be randomly identified as one or the other, with possibly non-IID misclassification error. We leave this for future work.

Another interesting avenue for future research would be exploring the implications of this model family for *both* labor market flows and the wages, for example, obtaining the wage data in a similar manner as Haefke, Sonntag, and Van Rens (2013). This would require restrictions on the stochastic process x and the wage function $w(x)$ and the value of unemployment $b(x)$.

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A Guide to the online appendix

The online appendix, available at

http://ihs.ac.at/~tpapp/structure-of-labor-market-flows-appendix.zip

contains all the proofs for the paper, including those which are omitted from the main text but also those which are presented with skipping trivial steps, coded in the symbolic algebra language Maxima (2014), which is available for free under the GNU General Public License (GPL). Use either the standard interface or a GUI like `wxMaxima`[22] to step through the proofs. The files are organized as follows:

file	content
`common.mac`	common setup for all derivations
`benchmark_model.mac`	derivations in Section 2, also loaded for comparison for other models
`data.mac`	data from various paper (cf Section 3)
`state_dependent_separations_model.mac`	derivations for Section 4.2
`permanently_inactive.mac`	derivations for Section 4.3
`learning_on_the_job.mac`	derivations for Section 4.4
`misclassification.mac`	derivations for Section 4.5
`calculations.r` (in R)	calculations for the literature summary graph and measurement error (Section 3), and for misclassification (Section 4.5)

All of these files load other files when necessary, and thus may be examined individually.

[22] http://andrejv.github.io/wxmaxima/